

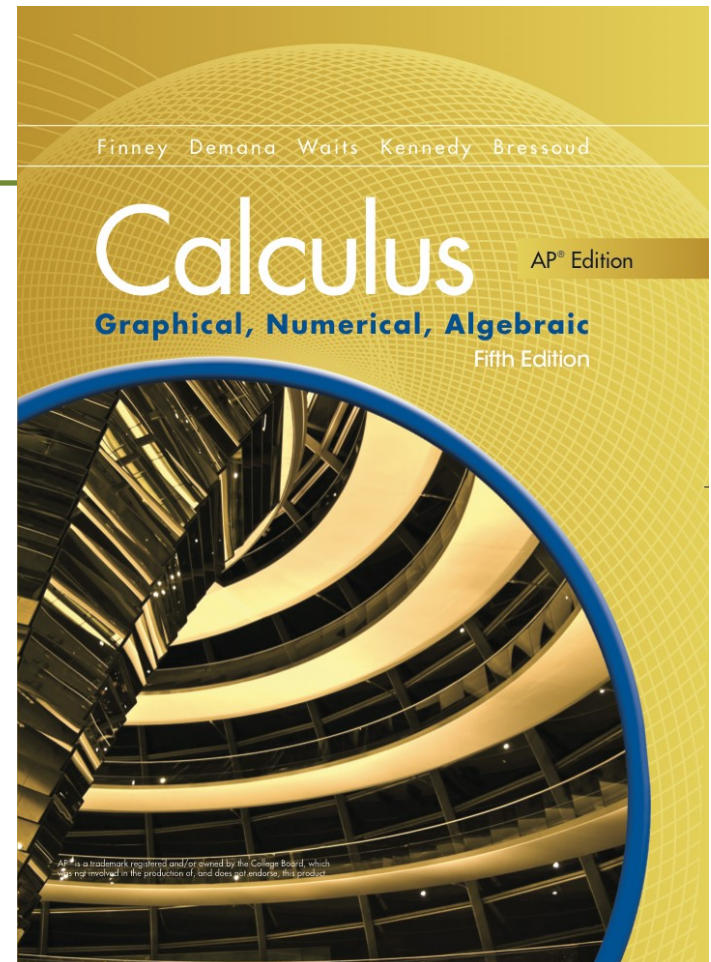
Chapter 6

Applications of Derivatives



Section 6.1

Estimating with Finite Sums



Quick Review

1. A train travels at 80 mph for 4 hours. How far does it travel?
2. Beginning at a standstill, a car maintains a constant acceleration of 5 ft/sec^2 for 10 seconds. What is its velocity after 10 seconds?
3. A pump working at 100 gallons/minute pumps for two hours. How many gallons are pumped?
4. At 8:00pm, the temperature began dropping at a rate of 1 degree Celcius per hour. Twelve hours later it began rising at a rate of 1.5 degrees per hour for six hours. What was the net change in temperature over the 18-hour period?

Quick Review Solutions

1. A train travels at 80 mph for 4 hours. How far does it travel? 320 miles
2. Beginning at a standstill, a car maintains a constant acceleration of 5 ft/sec^2 for 10 seconds. What is its velocity after 10 seconds? 50 ft/s
3. A pump working at 100 gallons/minute pumps for two hours. How many gallons are pumped? 12,000 gallons
4. At 8:00pm, the temperature began dropping at a rate of 1 degree Celcius per hour. Twelve hours later it began rising at a rate of 1.5 degrees per hour for six hours. What was the net change in temperature over the 18-hour period? 3°

What you'll learn about

- Distance as area under the velocity curve
- Accumulation as area under the curve representing rate of accumulation
- Estimation of area using rectangular approximation
- Estimation of volume using slices
- Use of left, right, and midpoint sums to approximate areas

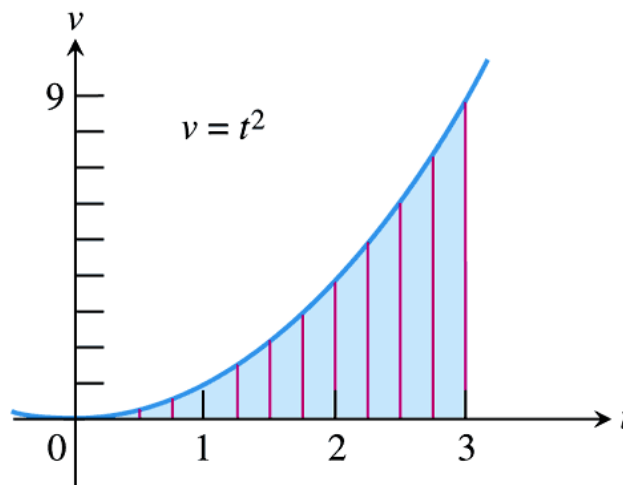
... and why

Learning about estimating with finite sums sets the foundation for understanding integral calculus.

Example Finding Distance Traveled when Velocity Varies

A particle starts at $x=0$ and moves along the x -axis with velocity $v(t) = t^2$ for time $t \geq 0$. Where is the particle at $t=3$?

Graph v and partition the time interval into subintervals of length Δt . If you use $\Delta t = 1/4$, you will have 12 subintervals. The area of each rectangle approximates the distance traveled over the subinterval. Adding all of the areas (distances) gives an approximation to the total area under the curve (total distance traveled) from $t=0$ to $t=3$.



Example Finding Distance Traveled when Velocity Varies

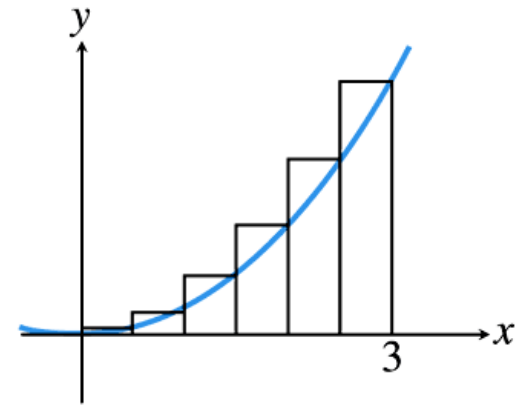
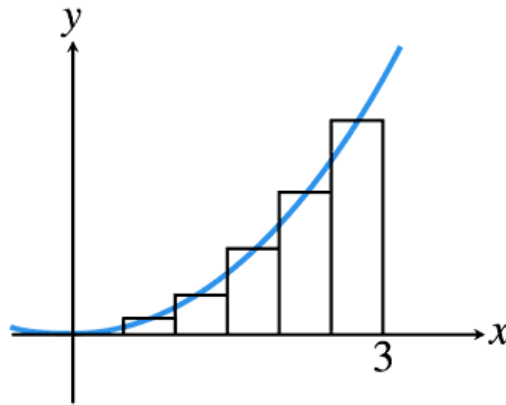
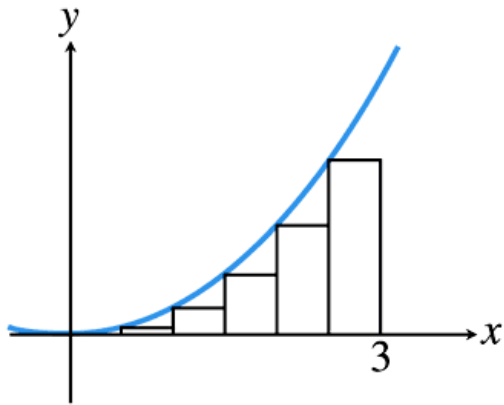
Table 6.1

Subinterval	$\left[0, \frac{1}{4}\right]$	$\left[\frac{1}{4}, \frac{1}{2}\right]$	$\left[\frac{1}{2}, \frac{3}{4}\right]$	$\left[\frac{3}{4}, 1\right]$
Midpoint m_i	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{5}{8}$	$\frac{7}{8}$
Height = $(m_i)^2$	$\frac{1}{64}$	$\frac{9}{64}$	$\frac{25}{64}$	$\frac{49}{64}$
Area = $(1/4)(m_i)^2$	$\frac{1}{256}$	$\frac{9}{256}$	$\frac{25}{256}$	$\frac{49}{256}$

Continuing in this manner, derive the area $(1/4)(m_i)^2$ for each subinterval and add them:

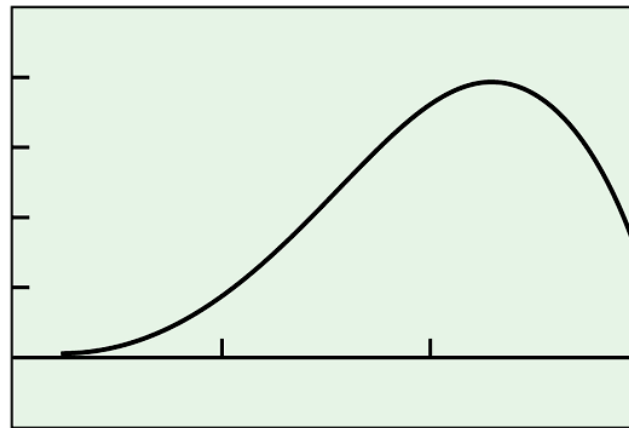
$$\frac{1}{256} + \frac{9}{256} + \frac{25}{256} + \frac{49}{256} + \frac{81}{256} + \frac{121}{256} + \frac{169}{256} + \frac{225}{256} + \frac{289}{256} + \frac{361}{256} + \frac{441}{256} + \frac{529}{256} = \frac{2300}{256} \approx 8.98$$

LRAM, MRAM, and RRAM approximations to the area under the graph of $y=x^2$ from $x=0$ to $x=3$



Example Estimating Area Under the Graph of a Nonnegative Function

Estimate the area under the graph of $f(x) = x^2 \sin x$ from $x=0$ to $x=3$.



$[0, 3]$ by $[-1, 5]$

Apply the RAM program (found in the *Technology Resource Manual* that accompanies this textbook).

Using 1000 subintervals, you find the left endpoint approximate area of 5.77476